Hypercomplex Type Algebras and PDE

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Abstract:

Traditionally one accept that differential equations are naturally defined on the field of real numbers $\mathbf{R}$. However, for instence, in the theory of harmonic oscilations complex numbers are admited. To my knowledge systematic exposition of differential equations on the field of complex numbers $\mathbf{C}$ is given in the book of Tricomi (1961).

The role of quaternions in relativistic physics is well known. Actually, it seems, relations with quantum mechanics are of interest. See, for instance, the book of S. L. Adler, Quaternionic Quantum Mechanics and Quantum Fields, Oxford Univ. Press (1995). The same is demonstrated with respect to some hypercomplex non-division algebras, like bicomplex and hyperbolic ones D. Rochon and S. Tremblay, (2004), A. Khrennikov, (2003).

The holomorphy CR-theory was developed in [D] and [ADMS] both in real and complex form. Here we present the basic equations.

In $\mathbf{C}_2 = \{\alpha = z + jw,\ j^2 = -1,\ z, w \in \mathbf{C}\}$

$$\frac{\partial f}{\partial z} = -\frac{\partial f}{\partial w},\ \frac{\partial f}{\partial w} = \frac{\partial f}{\partial z} \text{ (G. B. Price).}$$

In $\mathbf{C}(1, j) = \{\alpha = z + jw,\ j^2 = i,\ z, w \in \mathbf{C}\}$

$$\frac{\partial f}{\partial z} = i\frac{\partial f}{\partial w},\ \frac{\partial f}{\partial w} = \frac{\partial f}{\partial z} \text{ (S. D.).}$$

In the second case the so called double-complex Laplacian $\Delta f = \frac{\partial^2 f}{\partial z^2} + i\frac{\partial^2 f}{\partial w^2}$ attracted the attention of P. Popivanov [P] and L. Apostolova [A].

With the help of double-complex numbers one translate the real algebra $\mathbf{R}(1, j, j^2, j^3) = \{x = x_0 + jx_1 + j^2x_2 + j^3x_3\},\ j^4 = -1$ in the complex algebra $\mathbf{C}(1, j)$. In higher 2^n dimensions we have the same procedure, for instance, $\mathbf{R}(1, j, j^2, \ldots, j^7)$ is presented in the

form of $\mathbf{C}(1, j, j^2, j^3)$, setting $j^4 = i$. The later is not possible with the help of bicomplex numbers.

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