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UNIFORM STABILIZATION OF A QUASILINEAR MODEL IN HYPERBOLIC THERMOELASTICITY

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Resumo

We study dynamic elastic deformations of a quasilinear plate model of Timoshenko's type under thermal effects which are modeled by Cattaneo's law. The model is given by the following hyperbolic/hyperbolic coupled system

$$\begin{cases} u_{tt} - \mu\Delta u_{tt} + \Delta^2 u - M\left(\displaystyle\int_\Omega |\nabla u|^2 dx\right)\Delta u + \delta\Delta\theta_t = 0 \\ \tau\theta_{tt} - \rho\Delta\theta + \theta_t - \delta\Delta u_t = 0 \end{cases} \tag{1.1}$$

in $\Omega \times (0, +\infty)$. In (1.1), μ, δ, τ and ρ are positive constants. In the important thermoelastic physical case $n = 2$, $u = u(x,t)$ represents the vertical displacement of $x \in \Omega$ at time t. The function $\theta = \theta(x,t)$ denotes the temperature at $x \in \Omega$ at time t.

In (1.1) $M(s)$ is a continuous function $[0, +\infty) \to [0, +\infty)$ of class $C^1((0, +\infty))$ and non-decreasing for all $s \geq 0$.

We complement system (1.1) with Dirichlet boundary conditions

$$u = \Delta u = 0, \;\; \theta = 0 \;\text{ on }\; \partial\Omega \times (0, +\infty) \tag{1.2}$$

and initial conditions

$$u(x,0) = u_0(x),\; u_t(x,0) = u_1(x),\;\; \theta(x,0) = \theta_0(x),\; \theta_t(x,0) = \theta_1(x) \;\text{ in }\; \Omega. \tag{1.3}$$

We show global wellposedeness of the model and build a convenient Lyapunov function we prove uniform exponential stabilization of the total energy as time approaches infinity.