SOLVING A MINLP WITH CHANCE CONSTRAINT USING A ZHANG'S COPULA FAMILY

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Resumo

In recent years, the stochastic programming community have been witnessed a great development in optimization methods for dealing with stochastic programs with mixed-integer variables [2]. However, there are only few works on chance-constrained programming with mixed-integer variables [1, 3, 12, 13]. In this work, the problem of interest consists in nonsmooth convex mixed-integer nonlinear programs with chance constraints (CCMINLP). These class of problems for instance, can be solved by employing the outer-approximation technique. In general, OA algorithms require solving less MILP subproblems than extended cutting-plane algorithms [14], therefore the former class of methods is preferable than the latter one. This justifies why we have chosen the former class of methods to deal with problems of the type

$$
\begin{array}{lll}
\min_{(x,y)\in X\times Y} & f_0(x,y) & \\
\text{s.t.} & f_i(x,y) \leq 0, & i = 1,\ldots,m_f-1 \\
 & P[g(x,y) \geq \xi] \geq p, &
\end{array} \tag{1}
$$

where the functions $f_i : \mathcal{R}^{n_x} \times \mathcal{R}^{n_y} \to \mathcal{R}$, $i = 0,\ldots,m_f-1$, are convex but possibly nonsmooth, $g : \mathcal{R}^{n_x} \times \mathcal{R}^{n_y} \to \mathcal{R}^m$ is a concave function, $X \subset \mathcal{R}^{n_x}$ is a polyhedron,$Y \subset \mathcal{Z}^{n_y}$ contains only integer variables and both X and Y are compacts sets. Furthermore, $\xi \in \mathcal{R}^m$ is the random vector, P is the probability measure associated to the random vector ξ and $p \in (0,1)$ is a given parameter.

We assume that P is a $0-$concave distribution (thus P is $\alpha-$concave for all $\alpha \leq 0$). Some examples of distribution functions that satisfies the $0-$concavity property are the well-known multidimensional Normal, Log-normal, Gamma and Dirichlet distributions [9]. Under these assumptions, the following function is convex

$$
f_{m_f}(x,y) = \log(p) - \log(P[g(x,y) \geq \xi]). \tag{2}
$$

As a result, (1) is a convex (but possibly nonsmooth) MINLP problem fitting usually notation:

$$
f_{\min} := \min_{(x,y)\in X\times Y} f_0(x,y) \ \text{ s.t. } \ f_i(x,y) \leq 0\,,\ i \in \mathcal{I}_c := \{1,\ldots,m_f\}\,. \tag{3}
$$

Due to the probability function $P[g(x,y) \geq \xi]$, evaluating the constraint (2) and computing its subgradient is a difficult task: for instance, if P follows a multivariate normal distribution, computing a subgradient of $P[g(x,y) \geq \xi]$ requires numerically solving m integrals of dimension $m-1$. If the dimension m of ξ is too large, then creating a cut for function $\log(p) - \log(P[g(x,y) \geq \xi])$ is computationally challenging. In this situation, it makes sense to replace the probability measure by a simpler function.

In this manner, this work proposes to approximate the hard chance constraint $P[g(x,y) \geq \xi] \geq p$ by a copula $\mathcal{C}$:

$$
\mathcal{C}(F_{\xi_1}(g_1(x,y)), F_{\xi_2}(g_2(x,y)), \ldots, F_{\xi_m}(g_m(x,y))) \geq p. \tag{4}
$$

In addition to the difficulties present in MINLP models, we recall that the constraint functi
(4) can be nondifferentiable. To solve Problem (3) we use the outer-approximation algorith
This work is joint with Welington de Oliveira.

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